Sample Question Paper

Name of Programme : B.A / B.Sc. Mathematics
Semester : II
Paper Code : CMA-106
Paper Title : Vector Analysis and Solid Geometry
Full Marks : 80
Pass Marks : 35 **Duration : 3 Hours**

The figures in the margin indicate full marks for the questions
Answer all the questions:

1. Choose and rewrite the correct answer for each of the following: 1×4=4

a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are non-zero vectors, then $(\vec{a}\times\vec{c})\times(\vec{b}\times\vec{a})$ lies in the plane

(i) $\vec{a}$ and $\vec{b}$ (ii) $\vec{a}$ and $\vec{d}$
(iii) $\vec{a}$ and $\vec{c}$ (iv) $\vec{c}$ and $\vec{d}$

b) Given that $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{a}$ is any constant vector. Then curl $(\vec{r}\times\vec{a})$ is

(i) $\vec{a}$ (ii) $-\vec{a}$
(iii) $2\vec{a}$ (iv) $-2\vec{a}$

c) The equation $3x^2 - 4y^2 - 2z^2 = 7$ represents

(i) an ellipsoid (ii) a hyperboloid of one sheet
(iii) a hyperboloid of two sheet (iv) a hyperboloid paraboloid

d) The condition that the plane $lx + my + nz = p$ is a tangent plane to the sphere $x^2 + y^2 + z^2 = a^2$ is

(i) $p^2(l^2 + m^2 + n^2) = a^2$ (ii) $a^2(l^2 + m^2 + n^2) = p^2$
(iii) $a(l^2 + m^2 + n^2) = p^2$ (iv) $p(l^2 + m^2 + n^2) = a^2$

2. Write very short answer for each of the following questions: 1×5=5

a) Define the gradient of a scalar point function.
b) State Gauss divergence theorem.
c) Write the equation of the sphere described on the joint of the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ as diameter.
d) Define a right circular cylinder.
e) Name the central conicoid represented by $ax^2 + by^2 = 2z$ when the constants a and b are of opposite signs.

3. Answer the following questions: **3×5=15**

a) If $\phi = 2x^3y^2z^2$, then find the value of div.(grad ϕ) at the point $(1,-1,1)$.

b) Find the equation of the cone where vertex is (α,β,γ) and the guiding curve is $z=0$, $ax^2+by^2=1$.

c) Find the equation of the sphere passing through the origin and the points $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$.

d Find the generator of the paraboloid $4x^2-y^2=8z$ which pass through the point $(3,-2,4)$.

e) Find the locus of the centres of sections of the conicoid $ax^2+by^2+cz^2=1$ which touch the conicoid $Ax^2+By^2+Cz^2=1$.

4. Answer the following questions: **4×5=20**

a) Evaluate by Green's theorem

$$\oint_c \{(\cos x \sin y - xy)dx + \sin x \cos y dy\},$$

where C is the circle $x^2+y^2=1$ in the xy-plane described in the positive sense.

b) Prove that every section of a right circular cone by a plane perpendicular to its axis is a circle.

c) Prove that the enveloping cylinder of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ whose generators are parallel to the line $\frac{x}{0}=\frac{y}{\sqrt{a^2-b^2}}=\frac{z}{c}$ meet the plane $z=0$ is circle.

d) Find the equation of the sphere which touches the sphere $x^2+y^2+z^2=21$ at the point $(1,-2,4)$ and passes through the point $(3,4,0)$.

e) Prove that the equation of the right circular cylinder whose axis is $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ and radius r is $(l^2+m^2+n^2)(x^2+y^2+z^2-r^2)=(lx+my+nz)^2$.

5. Answer any two of the following questions: **6×2=12**

a) Show that the area bounded by a simple closed curve C is given by $\oint_C (xdy - ydx)$. Hence obtain the area of ellipse $x=a\cos t$, $y=b\sin t$.

b) Verify the divergence theorem for the vector function

$$\vec{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$$

taken over the rectangular parallelepiped

$$0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c.$$

c) Verify Stoke's theorem for

$$\vec{F} = xy\hat{i} + xy^2\hat{j}$$

integrated around the square vertices $(1,0,0)$, $(1,1,0)$, $(0,1,0)$ and $(0,0,0)$ where $\hat{i}$ and $\hat{j}$ are unit vectors along $x-$axis and $y-$axis respectively.

6. Answer any two of the following questions: 6×2=12

a) Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in two perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$.

b) Find the equation of the enveloping cylinder of the surface given by $ax^2+by^2+cz^2=1$, the generators being parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$.

c) Find the equation of the tangent plane to the paraboloid $ax^2+by^2=2z$ parallel to $lx+my+nz=0$.

7. Answer any two of the following questions: 6×2=12

a) Prove that through any point in space, in general, six normals can be drawn to any central conicoid.

b) The section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ with P as vertex by the plane $z=0$ is a circle. Find the locus of P.

c) Find the equations of the pair of tangent planes to the conicoid $ax^2+by^2+cz^2=1$ which passess through the line $u=lx+my+nz-p=0$ and $u'=l'x+m'y+n'z-p'=0$.

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